

Coupling constants of bottom (charmed) mesons with pion from three point QCD sum rules

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Abstract

In this article, the three point QCD sum rules is used to compute the strong coupling constants of vertices containing the strange bottomed (charmed) mesons with pion. The coupling constants are calculated, when both the bottom (charm) and pion states are off-shell. A comparison of the obtained results of coupling constants with the existing predictions is also made.

Key words: strong coupling constant, meson, QCD sum rules, bottom, charm.

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I. INTRODUCTION

During last ten years, there have been numerous published research articles devoted to the precise determination of the strong form factors and coupling constants of meson vertices via QCD sum rules (QCDSR) [1]. QCDSR formalism have also been successfully used to study some of the "exotic" mesons made of quark-gluon hybrid ($q\bar{q}g$), tetraquark states ($q\bar{q}q\bar{q}$), molecular states of two ordinary mesons, glueballs and many others [2]. Coupling constants can provide a real possibility for studying the nature of the bottomed and charmed pseudoscalar and axial vector mesons. More accurate determination of these coupling constants play an important role in understanding of the final states interactions in the hadronic decays of the heavy mesons. Our knowledge of the form factors in hadronic vertices is of crucial importance to estimate hadronic amplitudes when hadronic degrees of freedom are used. When all of the particles in a hadronic vertex are on mass-shell, the effective fields of the hadrons describe point-like physics. However, when at least one of the particles in the vertex is off-shell, the finite size effects of the hadrons become important. The following coupling constants have been determined by different research groups: $D^*D\pi$ [3, 4], $DD\rho$ [5], $D^*D\rho$ [6], $D^*D^*\rho$ [7], DDJ/ψ [8], D^*DJ/ψ [9], D^*D^*J/ψ [10], D_sD^*K , D_s^*DK [11], $DD\omega$ [12] and $VD_{s0}^*D_{s0}^*$, VD_sD_s , $VD_s^*D_s^*$ and $VD_{s1}D_{s1}$ [13], in the framework of three point QCD sum rules. It is very important to know the precise functional form of the form factors in these vertices and even to know how this form changes when one or the other (or both) mesons are off-shell [13].

In this review, we focus on the method of three point QCD sum rules to calculate, the strong form factors and coupling constants associated with the $B_1B^*\pi$, $B_1B_0\pi$, $B_1B_1\pi$, $D_1D^*\pi$, $D_1D_0\pi$ and $D_1D_1\pi$ vertices, for both the bottom (charm) and pion states being off-shell. The three point correlation function is investigated in two phenomenological and theoretical sides. In physical or phenomenological part, the representation is in terms of hadronic degrees of freedom which is responsible for the introduction of the form factors, decay constants and masses. In QCD or theoretical part, which consists of two, perturbative and non-perturbative contributions (In the present work the calculations contributing the quark-quark and quark-gluon condensate diagrams are considered as non-perturbative effects), we evaluate the correlation function in quark-gluon language and in terms of QCD degrees of freedom such as, quark condensate, gluon condensate, etc, by the help of the Wil-

son operator product expansion(OPE). Equating two sides and applying the double Borel transformations, with respect to the momentum of the initial and final states, to suppress the contribution of the higher states and continuum, the strong form factors are estimated.

The outline of the paper is as follows. In section II, by introducing the sufficient correlation functions, we obtain QCD sum rules for the strong coupling constant of the considered $B_1 B^* \pi$, $B_1 B_0 \pi$ and $B_1 B_1 \pi$ vertices. With the necessary changes in quarks, we can easily apply the same calculations to the $D_1 D^* \pi$, $D_1 D_0 \pi$ and $D_1 D_1 \pi$ vertices. In obtaining the sum rules for physical quantities, both light quark-quark and light quark-gluon condensate diagrams are considered as non-perturbative contributions. In section III, the obtained sum rules for the considered strong coupling constants are numerically analysed. We will obtain the numerical values for each coupling constant when both the bottom (charm) and pion states are off-shell. Then taking the average of the two off-shell cases, we will obtain final numerical values for each coupling constant. In this section, we also compare our results with the existing predictions of the other works.

II. THE THREE POINT QCD SUM RULES METHOD

In order to evaluate the strong coupling constants, it is necessary to know the effective Lagrangians of the interaction which, for the vertices $B_1 B^* \pi$, $B_1 B_0 \pi$ and $B_1 B_1 \pi$, are[14, 15]:

$$\begin{aligned}\mathcal{L}_{B_1 B^* \pi} &= g_{B_1 B^* \pi} B_1^\alpha (\pi^+ B_\alpha^{*-} - \pi^- B_\alpha^{*+}), \\ \mathcal{L}_{B_1 B_0 \pi} &= i g_{B_1 B_0 \pi} B_1^\alpha (B_0^- \partial_\alpha \pi^+ - \partial_\alpha B_0^- \pi^+) + H.c., \\ \mathcal{L}_{B_1 B_1 \pi} &= -g_{B_1 B_1 \pi} \epsilon^{\alpha\beta\gamma\sigma} \partial_\alpha B_{1\beta} (\partial_\gamma B_{1\sigma}^+ \pi^- + \pi^+ \partial_\gamma B_{1\sigma}^-),\end{aligned}\tag{1}$$

From these Lagrangians, we can extract elements associated with the $B_1 B^* \pi$, $B_1 B_0 \pi$ and $B_1 B_1 \pi$ momentum dependent vertices, that can be written in terms of the form factors:

$$\begin{aligned}\langle B_1(p', \epsilon') | B^*(p, \epsilon) \pi(q) \rangle &= g_{B_1 B^* \pi}(q^2) (\epsilon' \cdot \epsilon) \frac{p \cdot q}{m_{B_1}}, \\ \langle B_1(p', \epsilon') | B_0(p) \pi(q) \rangle &= g_{B_1 B_0 \pi}(q^2) \epsilon' \cdot q, \\ \langle B_1(p', \epsilon') | B_1(p, \epsilon) \pi(q) \rangle &= i g_{B_1 B_1 \pi}(q^2) \epsilon^{\alpha\beta\gamma\sigma} \epsilon'_\gamma(p') \epsilon_\sigma(p) p'_\beta q_\alpha,\end{aligned}\tag{2}$$

where p and p' are the four momentum of the initial and final mesons and $q = p' - p$, ϵ and ϵ' are the polarization vector of the B^* and B_1 mesons. We study the strong coupling

constants $B_1 B^* \pi$, $B_1 B_0 \pi$ and $B_1 B_1 \pi$ vertices when both π and $B^*[B_0(B_1)]$ can be off-shell. The interpolating currents $j^\pi = \bar{q}\gamma_5 q$, $j^{B_0} = \bar{q}Q$, $j_\nu^{B^*} = \bar{q}\gamma_\nu Q$ and $j_\mu^{B_1} = \bar{q}\gamma_\mu\gamma_5 Q$ are interpolating currents of π , B_0 , B^* , B_1 mesons, respectively with q being the up or down and Q being the heavy quark fields. We write the three-point correlation function associated with the $B_1 B^* \pi$, $B_1 B_0 \pi$ and $B_1 B_1 \pi$ vertices. For the off-shell $B^*[B_0(B_1)]$ meson, Fig.1 (left), these correlation functions are given by:

$$\Pi_{\mu\nu}^{B^*}(p, p') = i^2 \int d^4x d^4y e^{i(p'x - py)} \langle 0 | \mathcal{T} \{ j_\mu^{B_1}(x) j_\nu^{B^* \dagger}(0) j^{\pi \dagger}(y) \} | 0 \rangle, \quad (3)$$

$$\Pi_\mu^{B_0}(p, p') = i^2 \int d^4x d^4y e^{i(p'x - py)} \langle 0 | \mathcal{T} \{ j_\mu^{B_1}(x) j^{B_0 \dagger}(0) j^{\pi \dagger}(y) \} | 0 \rangle, \quad (4)$$

$$\Pi_{\mu\nu}^{B_1}(p, p') = i^2 \int d^4x d^4y e^{i(p'x - py)} \langle 0 | \mathcal{T} \{ j_\mu^{B_1}(x) j_\nu^{B_1 \dagger}(0) j^{\pi \dagger}(y) \} | 0 \rangle, \quad (5)$$

and for the off-shell π meson, Fig.1 (right), these quantities are:

$$\Pi_{\mu\nu}^\pi(p, p') = i^2 \int d^4x d^4y e^{i(p'x - py)} \langle 0 | \mathcal{T} \{ j_\mu^{B_1}(x) j^{\pi \dagger}(0) j_\nu^{B^* \dagger}(y) \} | 0 \rangle, \quad (6)$$

$$\Pi_\mu^\pi(p, p') = i^2 \int d^4x d^4y e^{i(p'x - py)} \langle 0 | \mathcal{T} \{ j_\mu^{B_1}(x) j^{\pi \dagger}(0) j^{B_0 \dagger}(y) \} | 0 \rangle, \quad (7)$$

$$\Pi_{\mu\nu}^\pi(p, p') = i^2 \int d^4x d^4y e^{i(p'x - py)} \langle 0 | \mathcal{T} \{ j_\mu^{B_1}(x) j^{\pi \dagger}(0) j_\nu^{B_1 \dagger}(y) \} | 0 \rangle, \quad (8)$$

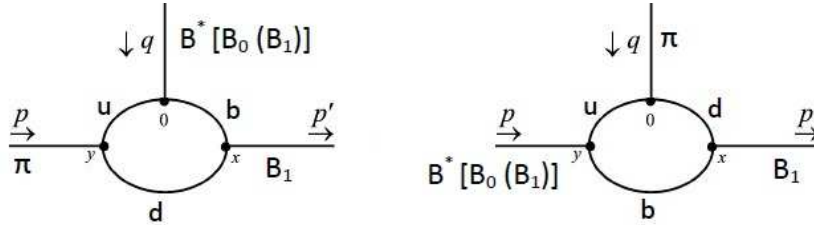


FIG. 1: perturbative diagrams for off-shell bottom (left) and off-shell pion (right).

Correlation function in (Eqs. (3 - 8)) in the OPE and in the phenomenological side can be written in terms of several tensor structures. We can write a sum rule to find the coefficients of each structure, leading to as many sum rules as structures. In principle all the structures should yield the same final results but, the truncation of the OPE changes different structures in different ways. Therefore some structures lead to sum rules which are more stable. In the simplest cases, such as in the $B_1 B^* \pi$ vertex, we have five structures

$g_{\mu\nu}$, $p_\mu p_\nu$, $p_\mu p'_\nu$, $p'_\mu p_\nu$ and $p'_\mu p'_\nu$. We have selected the $g_{\mu\nu}$ structure. In this structure the quark condensate (the condensate of lower dimension) contributes in the case of bottom meson off-shell. We also did the calculations for the structure $p'_\mu p_\nu$ and the final results of both structures in predicting of $g_{\mu\nu}$ are the same for $g_{B_1 B^* \pi}$ and in the $B_1 B_0 \pi$ vertex, we have two structure p'_μ and p_μ . The two structures give the same result for $g_{B_1 B_0 \pi}$. We have chosen the p'_μ structure. In the $B_1 B_1 \pi$ vertex we have only one structure $\epsilon^{\alpha\beta\mu\nu} p_\alpha p'_\beta$ is written as:

$$\begin{aligned}\Pi_{\mu\nu}^{B^*(\pi)}(p^2, p'^2, q^2) &= (\Pi_{per}^{B^*(\pi)} + \Pi_{nonper}^{B^*(\pi)})g_{\mu\nu} + \dots, \\ \Pi_\mu^{B_0(\pi)}(p^2, p'^2, q^2) &= (\Pi_{per}^{B_0(\pi)} + \Pi_{nonper}^{B_0(\pi)})p'_\mu + \dots, \\ \Pi_{\mu\nu}^{B_1(\pi)}(p^2, p'^2, q^2) &= (\Pi_{per}^{B_1(\pi)} + \Pi_{nonper}^{B_1(\pi)})\epsilon^{\alpha\beta\mu\nu} p_\alpha p'_\beta\end{aligned}\quad (9)$$

where \dots denotes other structures and higher states.

The phenomenological side of the vertex function is obtained by considering the contribution of three complete sets of intermediate states with the same quantum number that should be inserted in Eqs.(3 - 8). We use the standard definitions for the decay constants f_M (f_π , f_{B_0} , f_{B^*} and f_{B_1}) and are given by:

$$\begin{aligned}\langle 0 | j^\pi | \pi(p) \rangle &= \frac{m_\pi^2 f_\pi}{m_u + m_d}, \\ \langle 0 | j^{B_0} | B_0(p) \rangle &= m_{B_0} f_{B_0}, \\ \langle 0 | j_\nu^{B^*} | B^*(p, \epsilon) \rangle &= m_{B^*} f_{B^*} \epsilon_\nu(p), \\ \langle 0 | j_\mu^{B_1} | B_1(p', \epsilon') \rangle &= m_{B_1} f_{B_1} \epsilon'_\mu(p'),\end{aligned}\quad (10)$$

The phenomenological part for the $g_{\mu\nu}$ structure associated to $B_1 B^* \pi$ vertex, when $B^*(\pi)$ is off-shell meson is as follow:

$$\Pi_{\mu\nu}^{B^*(\pi)} = -g_{B_1 B^* \pi}^{B^*(\pi)}(q^2) \frac{m_\pi^2 m_{B^*} f_\pi f_{B^*} f_{B_1} (m_{B_1}^2 - m_{\pi(B^*)}^2 - q^2)}{2(q^2 - m_{B^*(\pi)}^2)(p^2 - m_{\pi(B^*)}^2)(p'^2 - m_{B_1}^2)(m_u + m_d)} g_{\mu\nu} + h.r., \quad (11)$$

The phenomenological part for the p'_μ structure related to the $B_1 B_0 \pi$ vertex, when $B_0(\pi)$ is off-shell meson is:

$$\Pi_\mu^{B_0(\pi)} = -g_{B_1 B_0 \pi}^{B_0(\pi)}(q^2) \frac{m_\pi^2 m_{B_0} m_{B^*} f_\pi f_{B^*} f_{B_1} (m_{B_1}^2 + m_{\pi(B_0)}^2 - q^2)}{2(q^2 - m_{B_0(\pi)}^2)(p^2 - m_{\pi(B_0)}^2)(p'^2 - m_{B_1}^2)(m_u + m_d)} p'_\mu + h.r., \quad (12)$$

The phenomenological part for the $\epsilon^{\alpha\beta\mu\nu} p_\alpha p'_\beta$ structure related to the $B_1 B_1 \pi$ vertex, when $B_1(\pi)$ is off-shell meson is:

$$\Pi_{\mu\nu}^{B_1(\pi)} = -ig_{B_1 B_1 \pi}^{B_1(\pi)}(q^2) \frac{m_\pi^2 m_{B_1}^2 f_\pi f_{B_1}^2}{(q^2 - m_{B_1(\pi)}^2)(p^2 - m_{\pi(B_1)}^2)(p'^2 - m_{B_1}^2)(m_u + m_d)} \epsilon^{\alpha\beta\mu\nu} p_\alpha p'_\beta + h.r.,$$

(13)

In the Eqs.(11 - 13), h.r. represents the contributions of the higher states and continuum.

With the help of the operator product expansion (OPE) in Euclidean region, where $p^2, p'^2 \rightarrow -\infty$, we calculate the QCD side of the correlation function (Eqs. (3 - 8)) containing perturbative and non-perturbative parts. In practice, only the first few condensates contribute significantly, the most important ones being the 3-dimension, $\langle \bar{d}d \rangle$, and the 5-dimension, $\langle \bar{d}\sigma_{\alpha\beta}T^a G^{a\alpha\beta}d \rangle$, condensates. For each invariant structure, i, we can write

$$\begin{aligned} \Pi_i^{(theor)}(p^2, p'^2, q^2) = & -\frac{1}{4\pi^2} \int_{(m_d+m_b)^2}^{\infty} ds' \int_{s_{1(2)}}^{\infty} ds \frac{\rho_i(s, s', q^2)}{(s-p^2)(s'-p'^2)} \\ & + C_i^3 \langle \bar{d}d \rangle + C_i^5 \langle \bar{d}\sigma_{\alpha\beta}T^a G^{a\alpha\beta}d \rangle + \dots, \end{aligned} \quad (14)$$

where $\rho_i(s, s', q^2)$ is spectral density, C_i are the Wilson coefficients and $G^{a\alpha\beta}$ is the gluon field strength tensor. We take for the strange quark condensate $\langle \bar{d}d \rangle = -(0.24 \pm 0.01)^3 \text{ GeV}^3$ [16] and for the mixed quark-gluon condensate $\langle \bar{d}\sigma_{\alpha\beta}T^a G^{a\alpha\beta}d \rangle = m_0^2 \langle \bar{d}d \rangle$ with $m_0^2 = (0.8 \pm 0.2) \text{ GeV}^2$ [17].

Furthermore, we make the usual assumption that the contributions of higher resonances are well approximated by the perturbative expression

$$-\frac{1}{4\pi^2} \int_{s'_0}^{\infty} ds' \int_{s_0}^{\infty} ds \frac{\rho_i(s, s', q^2)}{(s-p^2)(s'-p'^2)}, \quad (15)$$

with appropriate continuum thresholds s_0 and s'_0 .

The Cutkosky's rule allows us to obtain the spectral densities of the correlation function for the Lorentz structures appearing in the correlation function. The leading contribution comes from the perturbative term, shown in Fig.1. As a result, the spectral densities are obtained to the double discontinuity in Eq.(15) for vertices that are given in Appendix A.

We proceed to calculate the non-perturbative contributions in the QCD side that contain the quark-quark and quark-gluon condensate. The quark-quark and quark-gluon condensate is considered for when the light quark is a spectator [18], Therefore only three important diagrams of dimension 3 and 5 remain from the non-perturbative part contributions when the bottom meson are off shell. These diagrams named quark-quark and quark-gluon condensate are depicted in Fig.2. For the pion off-shell, there is no quark-quark and quark-gluon condensate contribution.

After some straightforward calculations and applying the double Borel transformations with respect to the $p^2(p^2 \rightarrow M^2)$ and $p'^2(p'^2 \rightarrow M'^2)$ as:

$$\begin{aligned} B_{p^2}(M^2) \left(\frac{1}{p^2 - m_u^2} \right)^m &= \frac{(-1)^m}{\Gamma(m)} \frac{e^{-\frac{m_u^2}{M^2}}}{(M^2)^m}, \\ B_{p'^2}(M'^2) \left(\frac{1}{p'^2 - m_b^2} \right)^n &= \frac{(-1)^n}{\Gamma(n)} \frac{e^{-\frac{m_b^2}{M'^2}}}{(M'^2)^n}, \end{aligned} \quad (16)$$

where M^2 and M'^2 are the Borel parameters, the contribution of the quark-quark and quark-gluon condensate for the bottom meson off-shell case, are given by:

$$\Pi_{(non-per)}^{bottom} = \langle \bar{d}d \rangle \frac{C^{bottom}}{M^4 M'^4}, \quad (17)$$

The explicit expressions for $C_{B_1 B^* \pi [B_1 B_0 \pi (B_1 B_1 \pi)]}^{bottom}$ associated with the $B_1 B^* \pi$, $B_1 B_0 \pi$ and $B_1 B_1 \pi$ vertices are given in Appendix B.

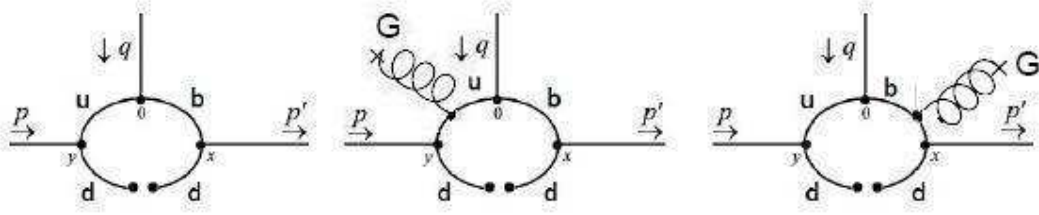


FIG. 2: Contribution of the quark-quark and quark-gluon condensate for the bottom off-shell.

The gluon-gluon condensate is considered when the heavy quark is a spectator [19], and the bottom mesons are off-shell, and there is no gluon-gluon condensate contribution. Our numerical analysis shows that the contribution of the non-perturbative part containing the quark-quark and quark-gluon diagrams is about 13% and the gluon-gluon contribution is about 4% of the total and the main contribution comes from the perturbative part of the strong form factors and we can ignore gluon-gluon contribution in our calculation.

The QCD sum rules for the strong form factors are obtained after performing the Borel transformation with respect to the variables $p^2(B_{p^2}(M^2))$ and $p'^2(B_{p'}^2(M'^2))$ on the physical (phenomenological) and QCD parts and equating these two representations of the correlations, we obtain the corresponding equations for the strong form factors as follows.

- For the $g_{B_1 B^* \pi}(Q^2)$ form factors:

$$g_{B_1 B^* \pi}^{B^*}(Q^2) = \frac{2(Q^2 + m_{B^*}^2)(m_u + m_d)}{m_\pi^2 m_{B^*} f_\pi f_{B^*} f_{B_1} (m_{B_1}^2 - m_\pi^2 + Q^2)} e^{\frac{m_\pi^2}{M^2}} e^{\frac{m_{B_1}^2}{M'^2}} \left\{ -\frac{1}{4\pi^2} \int_{(m_b + m_d)^2}^{s'_0} ds' \right\}$$

$$\times \int_{s_1}^{s_0} ds \rho^{B^*}(s, s', Q^2) e^{-\frac{s}{M^2}} e^{-\frac{s'}{M'^2}} + \langle d\bar{d} \rangle \frac{C_{B_1 B^* \pi}^{B^*}}{M^2 M'^2} \Big\}, \quad (18)$$

$$g_{B_1 B^* \pi}^\pi(Q^2) = \frac{2(Q^2 + m_\pi^2)(m_u + m_d)}{m_\pi^2 m_{B^*} f_\pi f_{B^*} f_{B_1} (m_{B_1}^2 - m_{B^*}^2 + Q^2)} e^{\frac{m_{B^*}^2}{M^2}} e^{\frac{m_{B_1}^2}{M'^2}} \left\{ -\frac{1}{4\pi^2} \int_{(m_b+m_d)^2}^{s'_0} ds' \right. \\ \left. \times \int_{s_2}^{s_0} ds \rho^\pi(s, s', Q^2) e^{-\frac{s}{M^2}} e^{-\frac{s'}{M'^2}} \right\}, \quad (19)$$

- For the $g_{B_1 B_0 \pi}(Q^2)$ form factors:

$$g_{B_1 B_0 \pi}^{B_0}(Q^2) = \frac{2(Q^2 + m_{B_0}^2)(m_u + m_d)}{m_\pi^2 m_{B_0} m_{B_1} f_\pi f_{B_0} f_{B_1} (m_{B_1}^2 + m_{B_0}^2 + Q^2)} e^{\frac{m_\pi^2}{M^2}} e^{\frac{m_{B_1}^2}{M'^2}} \left\{ -\frac{1}{4\pi^2} \int_{(m_b+m_d)^2}^{s'_0} ds' \right. \\ \left. \times \int_{s_1}^{s_0} ds \rho^{B_0}(s, s', Q^2) e^{-\frac{s}{M^2}} e^{-\frac{s'}{M'^2}} + \langle d\bar{d} \rangle \frac{C_{B_1 B_0 \pi}^{B_0}}{M^2 M'^2} \right\}, \quad (20)$$

$$g_{B_1 B_0 \pi}^\pi(Q^2) = \frac{2(Q^2 + m_\pi^2)(m_u + m_d)}{m_\pi^2 m_{B_0} m_{B_1} f_\pi f_{B_0} f_{B_1} (m_{B_1}^2 + m_{B_0}^2 + Q^2)} e^{\frac{m_{B_0}^2}{M^2}} e^{\frac{m_{B_1}^2}{M'^2}} \left\{ -\frac{1}{4\pi^2} \int_{(m_b+m_d)^2}^{s'_0} ds' \right. \\ \left. \times \int_{s_2}^{s_0} ds \rho^\pi(s, s', Q^2) e^{-\frac{s}{M^2}} e^{-\frac{s'}{M'^2}} \right\}, \quad (21)$$

- For the $g_{B_1 B_1 \pi}(Q^2)$ form factors:

$$g_{B_1 B_1 \pi}^{B_1}(Q^2) = -i \frac{(Q^2 + m_{B_1}^2)(m_u + m_d)}{m_\pi^2 m_{B_1}^2 f_\pi f_{B_1}^2} e^{\frac{m_\pi^2}{M^2}} e^{\frac{m_{B_1}^2}{M'^2}} \left\{ -\frac{1}{4\pi^2} \int_{(m_b+m_d)^2}^{s'_0} ds' \right. \\ \left. \times \int_{s_1}^{s_0} ds \rho^{B_1}(s, s', Q^2) e^{-\frac{s}{M^2}} e^{-\frac{s'}{M'^2}} + \langle d\bar{d} \rangle \frac{C_{B_1 B_1 \pi}^{B_1}}{M^2 M'^2} \right\}, \quad (22)$$

$$g_{B_1 B_1 \pi}^\pi(Q^2) = -i \frac{(Q^2 + m_\pi^2)(m_u + m_d)}{m_\pi^2 m_{B_1}^2 f_\pi f_{B_1}^2} e^{\frac{m_{B_1}^2}{M^2}} e^{\frac{m_{B_1}^2}{M'^2}} \left\{ -\frac{1}{4\pi^2} \int_{(m_b+m_d)^2}^{s'_0} ds' \right. \\ \left. \times \int_{s_2}^{s_0} ds \rho^\pi(s, s', Q^2) e^{-\frac{s}{M^2}} e^{-\frac{s'}{M'^2}} \right\}, \quad (23)$$

where $Q^2 = -q^2$, s_0 and s'_0 are the continuum thresholds and s_1 and s_2 are the lower limits of the integrals over s as:

$$s_{1(2)} = \frac{(m_{d(b)}^2 + q^2 - m_u^2 - s')(m_u^2 s' - q^2 m_{d(b)}^2)}{(m_u^2 - q^2)(m_{d(b)}^2 - s')}. \quad (24)$$

III. NUMERICAL ANALYSIS

In this section, the expressions of QCD sum rules obtained for the considered strong coupling constants are investigated. We choose the values of the meson and quark masses as: $m_u = (1.7-3.3) \text{ MeV}$, $m_d = (3.5-6.0) \text{ MeV}$, $m_\pi = 14 \text{ MeV}$, $m_{B^*} = 5.32 \text{ GeV}$, $m_{D^*} = 2.01 \text{ GeV}$, $m_{B_1} = 5.72 \text{ GeV}$, $m_{D_1} = 2.42 \text{ GeV}$, $m_{B_0} = 5.70 \text{ GeV}$, $m_{D_0} = 2.36 \text{ GeV}$. Also the leptonic decay constants used in this calculation are taken as: $f_\pi = 130.41 \text{ MeV}$ [20], $f_{B^*} = 238 \pm 10 \text{ MeV}$, $f_{D^*} = 340 \pm 12 \text{ MeV}$ [21], $f_{B_1} = 196.9 \pm 8.9 \text{ MeV}$, $f_{D_1} = 218.9 \pm 11.3 \text{ MeV}$ [22], $f_{B_0} = 280 \pm 31 \text{ MeV}$, $f_{D_0} = 334 \pm 8.6 \text{ MeV}$ [23]. For a comprehensive analysis of the strong coupling constants, we use the following values of the quark masses m_b and m_c in two sets: setI, $m_b(\overline{MS}) = 4.67 \text{ GeV}$ [24], $m_c = 1.26 \text{ GeV}$ [13, 25] and set II, $m_b(1S) = 4.19 \text{ GeV}$ [24], $m_c = 1.47 \text{ GeV}$ [13, 25].

The expressions for the strong form factors in Eqs.(18-23) should not depend on the Borel variables M^2 and M'^2 . Therefore, one has to work in a region where the approximations made are supposedly acceptable and where the result depends only moderately on the Borel variables. In this work we use the following relations between the Borel masses M^2 and M'^2 [5, 6]: $\frac{M^2}{M'^2} = \frac{m_\pi^2}{m_{B_1}^2 - m_b^2}$ for bottom meson off-shell and $M^2 = M'^2$ for pion meson off-shell. The values of the continuum thresholds $s_0 = (m + \Delta)^2$ and $s'_0 = (m_{B_1} + \Delta)^2$, where m is the π mass, for $B^*[B_0(B_1)]$ off-shell and the $B^*[B_0(B_1)]$ meson mass, for π off-shell and Δ varies between: $0.4 \text{ GeV} \leq \Delta \leq 1 \text{ GeV}$ [13, 25].

Using $\Delta = 0.7 \text{ GeV}$, $m_b = 4.67 \text{ GeV}$ and fixing $Q^2 = 1 \text{ GeV}^2$, We found a good stability of the sum rule in the interval $10 \text{ GeV}^2 \leq M^2 \leq 20 \text{ GeV}^2$ for the two cases of bottom and pion being off-shell. The dependence of the strong form factors $g_{B_1 B^* \pi}$, $g_{B_1 B_0 \pi}$ and $g_{B_1 B_1 \pi}$ on Borel mass parameters for off-shell bottom and pion mesons are shown in Fig.3. We have chosen the Borel mass to be $M^2 = 13 \text{ GeV}^2$. Having determined M^2 , we calculated the Q^2 dependence of the form factors. We present the results in Fig.4 for the $g_{B_1 B^* \pi}$, $g_{B_1 B_0 \pi}$ and $g_{B_1 B_1 \pi}$ vertices. In this figures, the small circles and boxes correspond to the form factors in the interval where the sum rule is valid. As it is seen, the form factors and their fit functions coincide together, well.

We discuss a difficulty inherent to the calculation of coupling constants with QCDSR. The solution of Eqs.(18-23) is numerical and restricted to a singularity-free region in the Q^2 axis, usually located in the space-like region. Therefore, in order to reach the pole position,

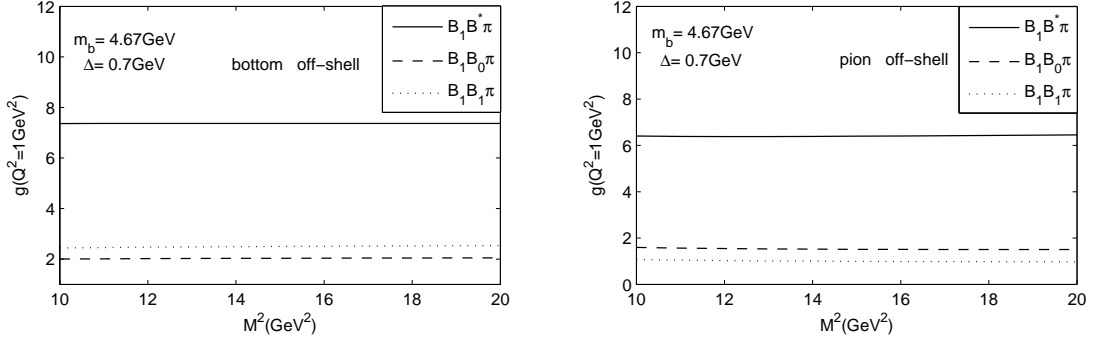


FIG. 3: The strong form factors $g_{B_1 B^* \pi}$, $g_{B_1 B_0 \pi}$ and $g_{B_1 B_1 \pi}$ as functions of the Borel mass parameter M^2 with for two cases bottom off-shell meson (left) and pion off-shell mesons (right).

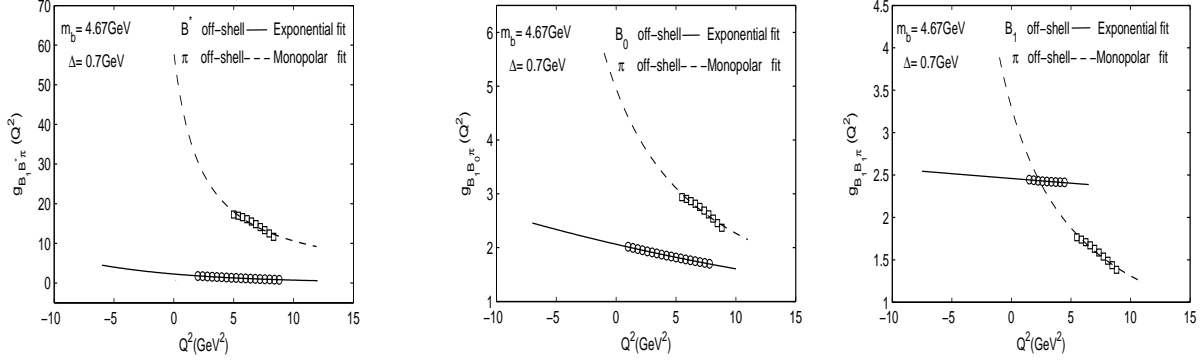


FIG. 4: The strong form factors $g_{B_1 B^* \pi}$, $g_{B_1 B_0 \pi}$ and $g_{B_1 B_1 \pi}$ on Q^2 for the bottom off-shell and the pion off-shell mesons. The small circles and boxes correspond to the form factors via the 3PSR calculations.

$Q^2 = -m_m^2$, we must fit the solution, by finding a function $g(Q^2)$ which is then extrapolated to the pole, yielding the coupling constant.

The uncertainties associated with the extrapolation procedure, for each vertex is minimized by performing the calculation twice, first putting one meson and then another meson off-shell, to obtain two form factors g^{bottom} and g^{pion} and equating these two functions at the respective poles. The superscripts in parenthesis indicate which meson is off-shell. In order to reduce the freedom in the extrapolation and constrain the form factor, we calculate and fit simultaneously the values of $g(Q^2)$ with the pion off-shell. We tried to fit our results to a monopole form, since this is often used for form factors [26].

For the off-shell pion meson, Our numerical calculations show that the sufficient

parametrization of the form factors with respect to Q^2 is:

$$g(Q^2) = \frac{A}{Q^2 + B}, \quad (25)$$

and for off-shell bottom meson the strong form factors can be fitted by the exponential fit function as given:

$$g(Q^2) = A e^{-Q^2/B}. \quad (26)$$

TABLE I: Parameters appearing in the fit functions for the $g_{B_1 B^* \pi}$, $g_{B_1 B_0 \pi}$ and $g_{B_1 B_1 \pi}$ vertices for $\Delta_1 = 0.7 \text{ GeV}$ and $m_b(\overline{MS}) = 4.67 \text{ GeV}$ (set I) and $m_b(1S) = 4.19 \text{ GeV}$ (set II).

	set I		set II	
Form factor	A	B	A	B
$g_{B_1 B^* \pi}^{B^*}$	2.26	8.73	4.35	11.56
$g_{B_1 B^* \pi}^{\pi}$	129.87	2.23	301.25	6.12
$g_{B_1 B_0 \pi}^{B_0}$	2.06	39.93	2.47	37.53
$g_{B_1 B_0 \pi}^{\pi}$	41.77	8.44	308.03	54.43
$g_{B_1 B^* \pi}^{B_1}$	2.46	219.04	2.59	132.90
$g_{B_1 B_1 \pi}^{\pi}$	21.77	6.60	205.82	60.51

TABLE II: The strong coupling constants $g_{B_1 B^* \pi}$, $g_{B_1 B_0 \pi}$ and $g_{B_1 B_1 \pi}$.

	set I		set II		
Coupling constant	bottom-off-sh	pion-off-sh	bottom-off-sh	pion-off-sh	Average
$g_{B_1 B^* \pi}$	57.63 ± 15.53	58.72 ± 15.43	50.32 ± 13.24	49.38 ± 14.26	54.01 ± 15.51
$g_{B_1 B_0 \pi}$	4.68 ± 1.44	4.96 ± 1.08	5.87 ± 1.34	5.66 ± 1.13	5.29 ± 1.40
$g_{B_1 B_1 \pi}(\text{GeV}^{-1})$	2.86 ± 0.43	3.31 ± 0.27	3.31 ± 0.25	3.89 ± 0.18	3.57 ± 0.53

The values of the parameters A and B are given in the Table I. We define the coupling constant as the value of the strong coupling form factor at $Q^2 = -m_m^2$ in the Eq. (25) and Eq. (26), where m_m is the mass of the off-shell meson. Considering the uncertainties result with the continuum threshold and uncertainties in the values of the other input

parameters, we obtain the average values of the strong coupling constants in different sets shown in Table II.

We can see that for the two cases considered here, the off-shell bottom and pion meson, give compatible results for the coupling constant.

The same method described in section II with little change in the containing perturbative and non-perturbative parts, where $\rho_{D_1 D^* \pi [D_1 D_0 \pi (D_1 D_1 \pi)]}^{charm(pion)} = \rho_{B_1 B^* \pi [B_1 B_0 \pi (B_1 B_1 \pi)]}^{bottom(pion)}|_{b \rightarrow c}$, $C_{D_1 D^* \pi [D_1 D_0 \pi (D_1 D_1 \pi)]}^{charm} = C_{B_1 B^* \pi [B_1 B_0 \pi (B_1 B_1 \pi)]}^{bottom}|_{b \rightarrow c}$, we can easily find similar results in Eqs.(18-23) for strong form factors $g_{D_1 D^* \pi}$, $g_{D_1 D_0 \pi}$ and $g_{D_1 D_1 \pi}$ and also use the following relations between the Borel masses M^2 and M'^2 : $\frac{M^2}{M'^2} = \frac{m_\pi^2}{m_{D_1}^2 - m_c^2}$ for charm meson off-shell and $M^2 = M'^2$ for pion meson off-shell. The values of the continuum thresholds $s_0 = (m + \Delta)^2$ and $s'_0 = (m_{D_1} + \Delta)^2$, where m is the π mass, for $D^*[D_0(D_1)]$ off-shell and the $D^*[D_0(D_1)]$ meson mass, for the π off-shell and Δ being between $0.4 \text{ GeV} \leq \Delta \leq 1 \text{ GeV}$.

Using $\Delta = 0.7 \text{ GeV}$, $m_c = 1.26 \text{ GeV}$ and fixing $Q^2 = 1 \text{ GeV}^2$, we found a good stability of the sum rule in the interval $7 \text{ GeV}^2 \leq M^2 \leq 17 \text{ GeV}^2$ for two cases of charm and pion off-shell. The dependence of the strong form factors $g_{D_1 D^* \pi}$, $g_{D_1 D_0 \pi}$ and $g_{D_1 D_1 \pi}$ on Borel mass parameters for the off-shell charm and pion mesons are shown in Fig.5. We have

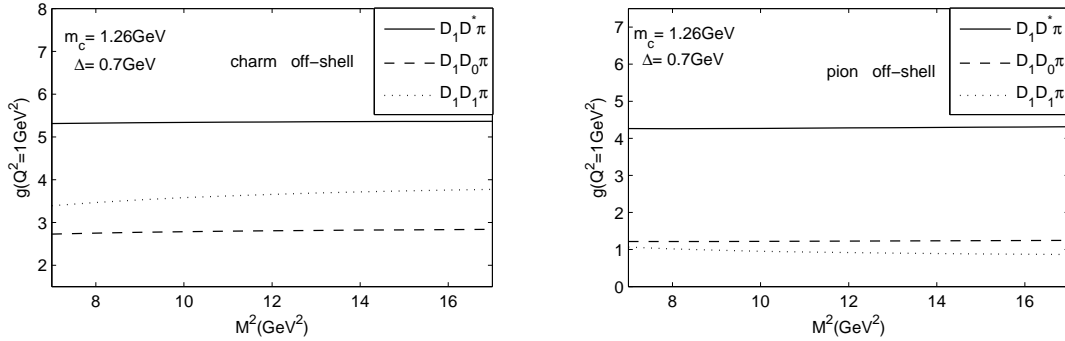


FIG. 5: The strong form factors $g_{D_1 D^* \pi}$, $g_{D_1 D_0 \pi}$ and $g_{D_1 D_1 \pi}$ as functions of the Borel mass parameter M^2 for the two cases of charm off-shell meson (left), and pion off-shell meson (right).

chosen the Borel mass to be $M^2 = 10 \text{ GeV}^2$. Having determined M^2 , we calculated the Q^2 dependence of the form factors. We present the results in Fig.6 for the $g_{D_1 D^* \pi}$, $g_{D_1 D_0 \pi}$ and $g_{D_1 D_1 \pi}$ vertices.

The dependence of the above strong form factors on Q^2 to the full physical region is estimated, using Eq.(25) and Eq.(26) for the pion and charm off-shell mesons, respectively. The values of the parameters A and B are given in the Table III.

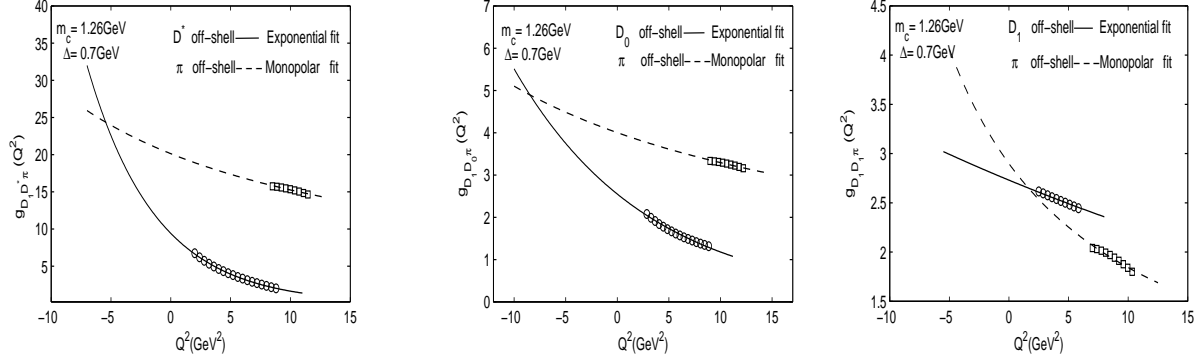


FIG. 6: The strong form factors $g_{D_1 D^* \pi}$, $g_{D_1 D_0 \pi}$ and $g_{D_1 D_1 \pi}$ dependence on Q^2 for the charm off-shell and the pion off-shell mesons. The small circles and boxes correspond to the form factors via the 3PSR calculations.

Considering the uncertainties result with the continuum threshold and uncertainties in the values of the other input parameters, we obtain the average values of the strong coupling constants in different values of the different sets shown in Table IV.

TABLE III: Parameters appearing in the fit functions for the $g_{D_1 D^* \pi}$, $g_{D_1 D_0 \pi}$ and $g_{D_1 D_1 \pi}$ vertices for $\Delta_1 = 0.7 \text{ GeV}$ and $m_c = 1.26$ (set I) and $m_c = 1.47$ (set II).

	set I		set II	
Form factor	A	B	A	B
$g_{D_1 D^* \pi}^{D^*}$	9.41	5.72	9.58	5.83
$g_{D_1 D^* \pi}^{\pi}$	63.07	31.30	86.40	4.18
$g_{D_1 D_0 \pi}^{D_0}$	2.55	12.97	2.37	13.05
$g_{D_1 D_0 \pi}^{\pi}$	185.69	46.40	32.98	8.49
$g_{D_1 D^* \pi}^{D_1}$	2.75	49.54	2.21	14.40
$g_{D_1 D_1 \pi}^{\pi}$	50.54	17.44	13.79	3.92

In Table V we compare our obtained values, with the findings of others, previously calculated. From this Table we see that our result of the coupling constants is in a fair agreement with the calculations in refs.[27, 28, 30].

TABLE IV: The strong coupling constants $g_{D_1 D^* \pi}$, $g_{D_1 D_0 \pi}$ and $g_{D_1 D_1 \pi}$.

Coupling constant	set I		set II		Average
	charm-off-sh	pion-off-sh	charm-off-sh	pion-off-sh	
$g_{D_1 D^* \pi}$	19.07 ± 4.21	20.14 ± 4.49	19.16 ± 3.87	20.77 ± 3.92	19.78 ± 3.32
$g_{D_1 D_0 \pi}$	3.92 ± 0.93	4.03 ± 1.01	3.63 ± 0.84	3.89 ± 0.73	3.87 ± 0.86
$g_{D_1 D_1 \pi}(GeV^{-1})$	3.09 ± 0.63	2.90 ± 0.52	3.31 ± 0.54	3.54 ± 0.61	3.21 ± 0.49

TABLE V: Comparison of our results with the other published results. The results of Refs. [27, 29] are from light-cone QCD sum rules, the result from Ref.[28] is from the QCD sum rules and the short distance expansion, and the result of Ref.[30] is from the light-cone QCD sum rules in HQET.

	$g_{B_1 B^* \pi}$	$g_{B_1 B_0 \pi}$	$g_{B_1 B_1 \pi}(GeV^{-1})$	$g_{D_1 D^* \pi}$	$g_{D_1 D_0 \pi}$	$g_{D_1 D_1 \pi}(GeV^{-1})$
Our result	54.01 ± 15.51	5.29 ± 1.40	3.57 ± 0.53	19.78 ± 3.32	3.87 ± 0.86	3.21 ± 0.49
Ref.[27]	56 ± 15	5.39 ± 2.15	-	23 ± 5	3.43 ± 1.37	-
Ref.[28]	-	-	-	19.12 ± 2.42	-	2.59 ± 0.61
Ref.[29]	68.64 ± 8.58	-	-	12.10 ± 2.42	-	-
Ref.[30]	58.89 ± 9.81	4.73 ± 1.14	2.60 ± 0.60	-	-	-

IV. CONCLUSION

In this article, we analyzed the vertices $B_1 B^* \pi$, $B_1 B_0 \pi$, $B_1 B_1 \pi$, $D_1 D^* \pi$, $D_1 D_0 \pi$ and $D_1 D_1 \pi$ within the framework of the three point QCD sum rules approach in an unified way. The strong coupling constants could give useful information about strong interactions of the strange bottomed and strange charmed mesons and also are important ingredients for estimating the absorption cross section of the J/ψ by the π mesons.

Appendix A: PERTURBATIVE CONTRIBUTIONS

In this appendix, The perturbative contributions for the sum rules defined in Eqs.(18-23) are:

$$\rho_{B_1 B^* \pi}^{B^*(\pi)} = 4N_c I_0 k \left[2A \left(m_1 - m_{3(2)} \right) - m_1 m_2 m_3 + m_2 m_3^2 + m_3^3 - m_1 m_3^2 - \frac{\Delta}{2} (m_2 + m_3) \right]$$

$$\begin{aligned}
& + \frac{\Delta'}{2} (m_1 - m_3) + \frac{m_3 u}{2} \Big], \\
\rho_{B_1 B_0 \pi}^{B_0(\pi)} &= 4N_c I_0 \left[B_2 \left(m_2 m_3 - k m_1 m_2 + k m_1 m_3 - m_3^2 + \Delta - \frac{u}{2} \right) + k m_3^2 - m_3 m_1 - k \frac{\Delta}{2} \right], \\
\rho_{B_1 B_1 \pi}^{B_1(\pi)} &= 4i N_c I_0 [B_1 (m_3 - k m_1) + B_2 (m_2 + m_3) + m_3],
\end{aligned}$$

The explicit expressions of the coefficients in the spectral densities entering the sum rules are given as:

$$\begin{aligned}
I_0(s, s', q^2) &= \frac{1}{4\lambda^{\frac{1}{2}}(s, s', q^2)}, \\
\Delta &= (s + m_3^2 - m_1^2), \\
\Delta' &= (s' + m_3^2 - m_2^2), \\
u &= s + s' - q^2, \\
\lambda(s, s', q^2) &= s^2 + s'^2 + q^4 - 2s q^2 - 2s' q^2 - 2s s', \\
A &= -\frac{1}{2\lambda(s, s', q^2)} [4s s' m_3^2 - s \Delta'^2 - s' \Delta^2 - u^2 m_3^2 + u \Delta \Delta'], \\
B_1 &= \frac{1}{\lambda(s, s', q^2)} [2s' \Delta - \Delta' u], \\
B_2 &= \frac{1}{\lambda(s, s', q^2)} [2s \Delta' - \Delta u],
\end{aligned}$$

Where $k = 1$, $m_1 = m_u$, $m_2 = m_b$, $m_3 = m_d$ for bottom meson off-shell and $k = -1$, $m_1 = m_u$, $m_2 = m_d$, $m_3 = m_b$ for pion meson off-shell, $N_c = 3$ represents the color factor.

Appendix B: NON-PERTURBATIVE CONTRIBUTIONS

In this appendix, the explicit expressions of the coefficients of the quark-quark and quark-gluon condensate of the strong form factors for the vertices $B_1 B^* \pi$, $B_1 B_0 \pi$ and $B_1 B_1 \pi$ with applying the double Borel transformations are given.

$$\begin{aligned}
C_{B_1 B^* \pi}^{B^*} &= \left(\frac{7M^2 m_b^2 m_0^2}{24} - \frac{M^2 M'^2 m_0^2}{6} + \frac{M'^2 m_b^2 m_0^2}{8} - \frac{m_0^2 m_b^4}{8} - \frac{M^2 M'^2 m_b m_d}{4} + \frac{M^2 M'^2 m_b^2}{2} \right. \\
&\quad - \frac{M^2 m_b^3 m_d}{4} + \frac{M^2 m_b m_d q^2}{4} - \frac{M^2 m_b^2 m_d^2}{2} - \frac{M^2 m_0^2 m_b m_u}{4} - \frac{3M'^2 m_0^2 m_b m_u}{4} \\
&\quad - M^2 M'^2 m_b m_u + \frac{m_0^2 m_b^3 m_u}{4} + \frac{M'^2 m_0^2 m_b^3 m_u}{2M^2} - \frac{M^2 M'^2 m_d m_u}{4} + \frac{M^2 m_b^2 m_d m_u}{2} \\
&\quad \left. + \frac{M'^2 m_b^2 m_d m_u}{4} + \frac{M^2 m_b m_d^2 m_u}{2} + \frac{M'^2 m_b m_d^2 m_u}{2} - \frac{m_b^3 m_d^2 m_u}{2} + \frac{M^2 m_0^2 m_u^2}{24} \right)
\end{aligned}$$

$$\begin{aligned}
& + \frac{M'^2 m_0^2 m_u^2}{4} + \frac{M^2 M'^2 m_u^2}{2} - \frac{m_0^2 m_b^2 m_u^2}{4} - \frac{M'^2 m_b m_d m_u^2}{2} - \frac{M'^2 m_d^2 m_u^2}{2} + \frac{m_b^2 m_d^2 m_u^2}{2} \\
& + \frac{m_0^2 m_b m_u^3}{4} - \frac{M'^2 m_0^2 m_b m_u^3}{2M^2} + \frac{M'^2 m_d m_u^3}{4} - \frac{m_b m_d^2 m_u^3}{2} - \frac{M'^2 m_d m_u q^2}{4} - \frac{7M^2 m_0^2 q^2}{24} \\
& - \frac{3M'^2 m_0^2 q^2}{8} - \frac{M^2 M'^2 q^2}{2} + \frac{m_0^2 m_b^2 q^2}{4} + \frac{M^2 m_d^2 q^2}{2} + \frac{M'^2 m_d^2 q^2}{2} - \frac{m_b^2 m_d^2 q^2}{2} \\
& - \frac{m_0^2 m_b m_u q^2}{4} \Big) \times e^{-\frac{m_u^2}{M^2}} e^{-\frac{m_b^2}{M'^2}}, \\
C_{B_1 B_0 \pi}^{B_0} = & \left(\frac{M^2 m_0^2 m_b}{4} - \frac{M^2 m_b^2 m_d}{2} - \frac{M^2 m_b m_d^2}{2} - \frac{3m_0^2 M'^2 m_u}{4} - M^2 M'^2 m_u + \frac{m_0^2 m_b^2 m_u}{4} \right. \\
& - \frac{M^2 m_b m_d m_u}{2} + \frac{M^2 m_d^2 m_u}{2} - \frac{m_b^2 m_d^2 m_u}{2} - \frac{M^2 m_d m_u^2}{2} - \frac{M'^2 m_d m_u^2}{2} + \frac{M'^2 m_d^2 m_u}{2} \\
& \left. + \frac{m_0^2 m_u^3}{4} - \frac{m_d^2 m_u^3}{2} + \frac{M^2 m_d q^2}{2} - \frac{m_0^2 m_u q^2}{4} + \frac{m_d^2 m_u q^2}{2} \right) \times e^{-\frac{m_u^2}{M^2}} e^{-\frac{m_b^2}{M'^2}}, \\
C_{B_1 B_1 \pi}^{B_1} = & i \left(\frac{7m_0^2 M^2}{12} + \frac{3m_0^2 M'^2}{4} + M^2 M'^2 - \frac{m_0^2 m_b^2}{2} - \frac{M^2 m_b m_d}{2} - M^2 m_d^2 - M'^2 m_d^2 \right. \\
& \left. + \frac{M'^2 m_d m_u}{2} - \frac{m_0^2 m_u^2}{2} + \frac{m_0^2 q^2}{2} - m_d^2 q^2 + m_b^2 m_d^2 \right) \times e^{-\frac{m_u^2}{M^2}} e^{-\frac{m_b^2}{M'^2}},
\end{aligned}$$

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